Modeling Kinetics of Anaerobic Co-Digestion of Poultry Litter and Wheat Straw Mixed with Municipal Wastewater in a Continuously Mixed Digester with Biological Solid Recycle Using Batch Experimental Data

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The half-saturation rate coefficient and maximum rate constant in the Monod model, yield coefficient defined as the ratio of microbial mass to substrate mass, and endogenous decay coefficient are important kinetic parameters for design of anaerobic digestion. These parameters are usually determined from a continuous stable operation of anaerobic digestion, which is more difficult and complex than batch operation in laboratory scale. In this study, a novel method has been developed to determine those parameters from data of batch experiments. To verify this method, kinetics of batch anaerobic co-digestion of poultry litter and wheat straw mixed with municipal wastewater at three total solid (TS) levels (2, 4, and 8% TS) and 50% volatile solid (VS) of wheat straw (VSWS) were investigated. The results showed that the maximum specific methane volume (209 mL (initial g VS)\(^{-1}\)) was reached at 4% TS of 50% VSWS. Using the developed method, the kinetic parameters of endogenous decay coefficient, yield coefficient, maximum rate constant, and half-saturation coefficient were determined to be between 0.57 \((10^3\text{ d}^{-1})\), 0.00938 and 0.0644 g volatile suspended solid (VSS) (VS)\(^{-1}\), 1.394 and 13,797 d\(^{-1}\), and 1.6 \((10^8\text{ g})\), and 99,996 g. The kinetic parameters obtained were used to simulate kinetic behaviors of a continuous mixed digester with biological solid recycle. The simulated results showed that the dilution rate was very significant for methane volume produced, VS and VSS concentrations in digestion operation. The maximum methane volume could be predicted to be 3071 and 4152 mL for 2 and 4% TS, respectively.

Keywords: Anaerobic digestion; Continuously mixed digester; Endogenous decay; Kinetic model; Poultry litter; Yield coefficient

Introduction

Arkansas, as the second largest state of poultry production in the United States, generates about 1.3 billion metric tons of poultry litter annually (Scharbor, 2011). The traditional disposal method for poultry litter is land application, which has caused several environmental concerns, such as land availability, deterioration of soil quality resulting from buildup of phosphorus in the soil (Chastain et al., 2012; Shih, 1987), and contaminations of ground and surface water (Priyadarssan et al., 2004). Anaerobic digestion is an effective way for utilization of livestock manures to generate renewable energy (methane). It was reported until March, 2015 that 247 anaerobic digester systems (dairy 202, hog 39, mixed 8, poultry 7, and beef 8) were operating at commercial livestock farms in the United States (AgSTAR-EPA, 2015). Therefore, anaerobic digestion of poultry litter is considered an alternative to land application with the following advantages: (1) renewable gas fuel (bio-methane) production, which can offset the operating cost of anaerobic digestion process (Singh et al., 2010); (2) regulating the nutrient loads of poultry litter to soils (Kelleher et al., 2002); (3) elimination of the nuisance odors (Kelleher et al., 2002); and (4) reduction of pathogens in the digested effluent (Horan et al., 2004).

Although anaerobic digestion has those benefits, only a few digesters for treatment of poultry litter are in operation. One of the reasons for unpopular poultry litter digestion is that a massive water volume is required to form slurry in anaerobic digesters because poultry litter contains only about 15% moisture. Besides, unlike dairy and pig farms, poultry farms do not generate a great volume of washing wastewater, and use of tap water for dilution of poultry litter is neither feasible nor economical. Therefore, one of the options for digesting poultry litter is to find an alternative water source. Geographically, the poultry industry in Arkansas mainly concentrates in the northwest area of the state where wastewater from municipal wastewater treatment plants is readily available. For instance, there is a municipal wastewater plant located at Fayetteville, AR with a treatment capacity of an average daily flow of 38.9 million liters. The wastewater with low COD (around 460 mg L\(^{-1}\)) provides a potential water source for dilution of poultry litter. The energy in methane (high heating value 37.81 \((10^6\text{ J m}^{-3})\) and lower heating value 33.94 \((10^6\text{ J m}^{-3})\)
produced from anaerobic digestion can offset the operating cost of both aerobic and anaerobic treatments in the wastewater plant, which will reduce the utility expenses for the residents in the northwest Arkansan area.

Another issue with poultry litter for anaerobic digestion is its low carbon to nitrogen ratio, which often needs addition of external carbon-rich materials, such as wheat straw, to improve methane yield (co-digestion). Since Arkansas produces over 24 million bushels of wheat annually, beneficial use of the wheat straw, a renewable resource, should also be explored.

There are generally two methods for design of anaerobic digesters: (1) loading factors (Metcalf and Eddy, 1999) and (2) kinetic parameters (Lawrence, 1971). If the Monod model is applied for describing microbial growth, the required kinetic parameters for digester design are included in the following two ordinary differential Equations (1, 2):

\[
\frac{dS}{dt} = \frac{KSYX/S - KXS}{K_S + S}
\]

and

\[
\frac{dX}{dt} = \frac{YXS - KXS}{K_S + S} - k_dX
\]

where \(S\) is the substrate mass in the digester represented by the volatile solid (VS) in this study (g), \(X\) is the microbial mass represented by the volatile suspended solid (VSS) (g), \(K\) is the maximum rate constant in the Monod model (d\(^{-1}\)), \(K_S\) is the half-saturation rate coefficient in the Monod model (g), \(Y_{XS}\) is the yield coefficient of microbial mass to substrate mass (g VSS (g VS\(^{-1}\)), \(k_d\) is the endogenous decay coefficient (d\(^{-1}\)), and \(t\) is the digestion time (d). Among these kinetic parameters, the endogenous decay and yield coefficients in Equation (2) are usually determined by a linear plot of Equation (3) (i.e., \((S_0 - S)/(X - X_0)\)) against \(tX/(X - X_0)\),

\[
\frac{S_0 - S}{X - X_0} = \frac{k_d}{Y_{XS}} \frac{tX}{X - X_0} + \frac{Y_{XS}}{Y_{XS/S}}
\]

where \(S_0\) and \(X_0\) are the initial substrate and microbial mass (g), or other similar equations, such as

\[
\frac{1}{Y_{XS/S}} = \frac{k_d\theta}{Y_{XS}} + \frac{1}{Y_{XS/S}}
\]

where \(Y_{XS/S}\) and \(Y_{XS}\) are the observed and true yields of microbial mass to substrate mass (both with unit g VSS (g VS\(^{-1}\)), respectively, and \(\theta\) is the hydraulic retention time (d) (Metcalf and Eddy, 1999), from a continuous stable operation of anaerobic digestion, such as using a chemostat reactor, whereas the maximum rate constant and half-saturation rate coefficient in Equation (1) can be determined by another linear plot for Equation (5) when the endogenous decay and yield coefficients are obtained from Equation (3) or (4):

\[
\frac{1}{\nu} = \frac{K_S}{KY_{XS/S}S} + \frac{1}{KY_{XS/S}m} = \frac{K_S}{\nu_mS} + \frac{1}{\nu_m}
\]

where \(\nu\) (=\(Xdt/dS\)) and \(\nu_m\) (=\(KY_{XS/S}\)) are the specific substrate consumption rate and maximum substrate consumption rate (both with unit d g VSS (g VS\(^{-1}\)), respectively. Because continuous and stable operation of anaerobic digestion is more difficult and complex than batch operation in laboratory scale (Argaman et al., 2000), most experiments of anaerobic digestion, particularly involving solid waste digestion, are conducted in batch operation. Argaman et al (2000) successfully applied kinetic data from batch experiments to predict the performance of continuous stirred tank reactor (CSTR) systems for treatment of multicomponent industrial wastewater. Another barrier for applying Equations (3) and (4) is that it is often difficult to take a series of liquid samples from batch experiments of anaerobic digestion for measurement of VSS and VS without disruption of the anaerobic environment except at the initial and final times. Thus, how to estimate the endogenous decay and yield coefficients of anaerobic digestion from batch experimental data becomes a challenge.

Shen and Zhu (Shen and Zhu, 2016a) have developed a method to estimate the endogenous decay and yield coefficients from the experimental data of batch anaerobic digestion using numerical integration. But the other two kinetic parameters, i.e., the maximum rate constant and half-saturation rate coefficient, in the Monod model have not been determined in our previous study.

The objectives of this study were to (1) develop a novel method to determine the kinetic parameters of anaerobic digestion using the batch experimental data, such as methane volumes produced and the initial and final levels of VSS, (2) investigate the kinetics of batch anaerobic co-digestion of poultry litter and wheat straw mixed with municipal wastewater at different total solids (TS) levels, and (3) predict the VS, VSS, and methane volume changes with respect to the digestion time in a continuously mixed digester with biological solid recycle using the kinetic parameters obtained from batch experiments.

**Materials and Methods**

**Materials**

The poultry litter and wheat straw used in the experiments were ground in a Wiley mill and passed through a 20-mesh screen. The fractions of TS, VS (based on TS), and moisture content of these two materials were determined in our laboratory and presented in Table I. The properties of the used wastewater taking from the Fayetteville municipal wastewater treatment plant were also shown in Table I.

<table>
<thead>
<tr>
<th>Materials</th>
<th>Poultry litter</th>
<th>Wheat straw</th>
<th>Wastewater</th>
</tr>
</thead>
<tbody>
<tr>
<td>TS fraction</td>
<td>0.6662 ± 0.0464</td>
<td>0.9142 ± 0.0191</td>
<td></td>
</tr>
<tr>
<td>VS fraction</td>
<td>0.7085 ± 0.0462</td>
<td>0.9747 ± 0.0101</td>
<td></td>
</tr>
<tr>
<td>Moisture based on wet C/N ratio</td>
<td>0.2497 ± 0.0256</td>
<td>0.0941 ± 0.0228</td>
<td>6.82</td>
</tr>
<tr>
<td>COD mg L(^{-1})</td>
<td>458 ± 6.72</td>
<td></td>
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</tbody>
</table>
Experimental Methods

The experimental and analytical methods were described in our previous paper (Shen and Zhu, 2016b). Briefly, the co-digestion experiments of poultry litter and wheat straw were conducted in graduated 1000 mL flasks sealed with rubber stoppers and connected with either 500 mL or 1000 mL gas bags (Tedlar Bag, CEL scientific Corp.) at the top of the flasks, depending on gas volumes produced. There were valves between the flasks and gas bags to prevent air from entering into the flasks during gas bag replacement to maintain anaerobic environment in the digesters. Prior to the experiments, the anaerobic active slurry as inoculum was prepared for 25 days using 2% TS poultry litter with addition of half-dried active sludge (85% moisture content) taken from the Fayetteville municipal wastewater treatment plant and stored in the refrigerator. The poultry litter and wheat straw were mixed with 100 mL anaerobic activated slurry prepared in our laboratory and the wastewater taken from the Fayetteville wastewater treatment plant to form 2, 4, and 8% total dried solid suspensions (total volume 500 mL). The three mixtures of poultry litter and wheat straw (50% VS in total % VS, referred to as VSWS) were examined in co-digestion. The experiments were run in duplicate, and the average values were reported. Liquid samples in the initial and final stages of the experiments were taken from the suspensions for analysis. The suspensions were purged with pure nitrogen for 5 minutes to remove the air in the flask headspace prior to the experiments. The flasks containing the suspensions were kept at 37 °C in an incubator (Fisher Scientific Isotemp Standard Incubators 600 Series) and were manually shaken twice a day for about 30 seconds. Biogas samples were taken daily in the first two days and every three days thereafter until no biogas in the flasks was produced. Controlled digestions (100 mL activated slurry plus 400 mL wastewater without poultry litter and wheat straw) also were conducted so that the effect of activated slurry on digestion performance could be illustrated and eliminated.

Analytical Methods

Total solid and volatile solid were measured using the gravity method. Total and soluble CODs and free ammonia-nitrogen were analyzed using a DR 3900 Hach Spectrophotometer and a DRB 200 Hach Reactor following the manuals of the manufacturer and the standard methods (APHA, 1998). The biogas composition in the gas bags was analyzed using a Shimadzu Gas Chromatograph (GC 2014) equipped with a thermal conductivity detector (TCD) and a ShinCarbon column (Restek, length 2 m × ID 2 mm). The temperature of the column was kept at 40°C for 2 min, followed by increasing at a rate of 25°C per min until 150°C, and then held for 1 min. The temperatures for both the injector and FID was 250°C. The injected liquid sample volume was 0.006 mL. Biogas volumes produced from the digesters were measured using a wet gas meter (Model XMF-1, Shanghai Cixi Instrument Co., Ltd).

Routine statistical analysis was conducted for the models based on the parameters such as the sums of squares of residuals (SSres) and coefficients of determination (R²) to estimate the simulation effects, which are defined as follows:

\[ R^2 = 1 - \frac{SS_{res}}{SS_{tot}} \]  

\[ SS_{res} = \sum_{i=0}^{m} (P_i - p_i)^2 \]  

where \( P_i \) is the value of point \( i \) calculated from the models, \( p_i \) is the observed value of point \( i \), and \( SS_{tot} \) is the total sum of squares of deviations:

\[ SS_{tot} = \sum_{i=0}^{m} (\bar{p} - p_i)^2 \]  

where \( \bar{p} \) is the mean of the observed data, \( i \) is the index of observed points, and \( m \) is the sample size.

Results and Discussion

Kinetics of Co-Digestion of Poultry Litter and Wheat Straw

Figure 1(a) and b are the cumulative specific methane volume (mL (initial VS g)−1) and cumulative methane volume, respectively, with respect to the digestion time at 2, 4, and 8% TS of 50% VSWS during anaerobic co-digestion of poultry litter and wheat straw. The cumulative specific methane volumes in the three experiments generally increased with the digestion time. After 41 days, the maximum specific methane volumes for 2 and 4% TS were reached with no further increases, but that for 8% TS was still rising. At the end, the 4% TS in 50% VSWS co-digestion produced the maximum specific methane volume of 209 mL (g initial VS)−1, compared to the other two solids levels (2 and 8% TS (Figure 1(a))) having the final specific methane volumes of 162 and 102 mL (g initial VS)−1, respectively, while the corresponding maximum methane volumes were 3436, 1329, and 3478 mL (Figure 1(b)). The maximum specific methane volume achieved by 4% TS in the experiment may be attributed to its highest carbon to nitrogen ratio (39.9) among the three co-digestions (the C/N ratios: 24.3 and 14.7 for 2 and 8% TS, respectively) (Table II). Meanwhile, the VS removals were about 44.0, 84.0, and 72.6% for 2, 4, and 8% TS, respectively (Table II). The final concentration of total volatile acids increased from 0.162 to 1.85 mg L−1 with increasing TS percentage (Table II). There was one exponential increasing period of methane volume for each experiment occurred at the time.
periods of 8–32, 8–32, and 8–38 days for 2, 4, and 8% TS, respectively. Figure 1(c) is the methane fractions in the biogas of the three experiments with respect to digestion time, and the highest methane fractions were around 0.7 – 0.8 for all the treatments.

Table II. The properties of slurry of 50% VSWS before and after digestion

<table>
<thead>
<tr>
<th>TS (%)</th>
<th>2</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial pH</td>
<td>7.56</td>
<td>7.69</td>
<td>7.55</td>
</tr>
<tr>
<td>Final pH</td>
<td>7.9</td>
<td>7.7</td>
<td>7.53</td>
</tr>
<tr>
<td>Initial total COD (mg L(^{-1}))</td>
<td>82,620</td>
<td>165,240</td>
<td>371,337</td>
</tr>
<tr>
<td>Final total COD (mg L(^{-1}))</td>
<td>7458</td>
<td>19,480</td>
<td>40,060</td>
</tr>
<tr>
<td>Initial soluble COD (mg L(^{-1}))</td>
<td>20,085</td>
<td>40,170</td>
<td>90,272</td>
</tr>
<tr>
<td>Final soluble COD (mg L(^{-1}))</td>
<td>1815</td>
<td>4085</td>
<td>22,260</td>
</tr>
<tr>
<td>Ratio of C to N</td>
<td>24.3</td>
<td>39.9</td>
<td>14.7</td>
</tr>
<tr>
<td>Final NH(_3) (mg L(^{-1}))</td>
<td>1186</td>
<td>1410</td>
<td>9020</td>
</tr>
<tr>
<td>Final total acids (mg L(^{-1}))</td>
<td>0.162</td>
<td>0.523</td>
<td>1.85</td>
</tr>
<tr>
<td>Initial VSS (g L(^{-1}))</td>
<td>0.07185</td>
<td>0.3195</td>
<td>0.2056</td>
</tr>
<tr>
<td>Final VSS (g L(^{-1}))</td>
<td>0.2695</td>
<td>0.5565</td>
<td>0.685</td>
</tr>
<tr>
<td>Initial VS (g L(^{-1}))</td>
<td>8.2</td>
<td>16.41</td>
<td>36.92</td>
</tr>
<tr>
<td>Final VS (g L(^{-1}))</td>
<td>4.59</td>
<td>2.63</td>
<td>10.12</td>
</tr>
<tr>
<td>VS removal %</td>
<td>44.0</td>
<td>84.0</td>
<td>72.6</td>
</tr>
</tbody>
</table>

Determination of Endogenous Decay and Yield Coefficients in Entire Digestion Periods

The yield coefficients of methane to substrate, \(Y_{PS}\), are 368, 253, and 140 mL (g VS\(^{-1}\)) for 50% VSWS in 2, 4, and 8% TS, respectively, in the entire co-digestion periods (Table III). The increase of TS% led to the decrease of \(Y_{PS}\), which could be attributed to the larger mass transfer resistance for higher TS contents (Abbassi-Guendouz et al., 2012). It was interesting in noting that the order of methane yield coefficients to VS was different from that of the cumulative specific methane volumes because 4% TS, rather than 2% TS, produced the maximum cumulative specific methane volume of 209 mL (g initial VS\(^{-1}\)). This observation could be explained by the fact that the yield coefficient was obtained based on the consumed VS content, while the specific methane volume was based on the initial VS mass.

The observed yields (\(Y_{XSob}\) and \(Y_{XPob}\)) of VSS to VS, and VSS to methane in the entire co-digestion period were also shown in Table III. According to the definition of the observed
yield (Metcalf and Eddy, 1999), the carbon consumption by microbial growth and decay was not considered in both observed coefficients, i.e.,

$$Y_{X/Sob} = \frac{VSS_{fob} - VSS_{iob}}{VSS_{iob}}$$

$$Y_{X/Pob} = \frac{10^{-3}}{VSS_{fob}}$$

$$Y_{X/P} = \frac{10^{-3}}{VSS_{fob}}$$

$$Y_{X/S} = \frac{10^{-3}}{VSS_{fob}}$$

$$k_d (10^{-3}) d^{-1}$$

$$K (d^{-1})$$

$$K_S (g)$$

$$R^2$$

where the subscripts $i$, $f$, and $ob$ denote the initial, final, and observed data. According to the calculation procedure proposed by Shen and Zhu (2016a), the calculated endogenous decay coefficients in the entire co-digestion period were $1.2 \times 10^{-3}$, $0.57 \times 10^{-3}$, and $0.88 \times 10^{-3} d^{-1}$ for 2, 4, and 8% TS, respectively (Table III), and these values were much smaller than those found in aerobic wastewater treatment processes. For example, Chen et al. (2008) observed that the endogenous decay coefficient was between 0.065 and 0.069 $d^{-1}$ under high organic loading rates in sequencing batch reactors, and they attributed this to the much faster microbial growth in aerobic environment than in anaerobic environment. The high observed yields of VSS to VS (0.142 to 0.231) in aerobic environment were also observed in the experiments by Chen et al. (2008) compared to our results in anaerobic environment (0.00885–0.0622). Therefore, it is reasonable to state that the endogenous decay coefficients in anaerobic environment are much smaller than in aerobic environment. The values of endogenous decay coefficients in our study are also smaller than those used in the design of commercial wastewater treatment plants, which usually ranged from 0.01 to 0.05 $d^{-1}$ (Metcalf and Eddy, 1999). The reason is that the yield coefficient is a constant only in the microbial exponential growth period, but not in the entire co-digestion period. Our previous study demonstrated that the endogenous decay coefficients in the exponential periods were much larger than in the entire co-digestion periods (Shen and Zhu, 2016a). The yield coefficients, $Y_{X/S}$ of VSS to VS, in this study are also shown in Table III, which were obtained by multiplying $Y_{X/P}$ with $Y_{P/S}$ (= methane volume produced divided by VSS produced). The values of $Y_{X/S}$ are 0.0644, 0.0363, and $0.00938 g VSS (g VS)^{-1}$ for 2, 4, and 8% TS, respectively, the first two of which are in a typical range from 0.02 to 0.1 (Metcalf and Eddy, 1999). Furthermore, all the observed yield coefficients are less than the corresponding (non-observed or true) yield coefficients because of the influence of the microbial endogenous decay (Table III) (Metcalf and Eddy, 1999).

### Determination of the Maximum Rate Constant and Half-Saturation Rate Coefficients in the Monod Model

By means of the method developed by Shen and Zhu (2016a), the profiles of VSS (i.e., $X$) and VS (i.e., $S$) of 2, 4, and 8% TS in digestion were also determined (Figure 2), and these values were fitted to Equation (1) to determine the maximum rate constant and half-saturation rate coefficient using the non-linear regression program in MATLAB.

![Fig. 2. VSS and calculated VSS (referred to as Cal) (a), VS and calculated VS (b) in the entire periods at 2 (square), 4 (triangle), and 8 (circle) % TS of 50% VSWS with respect to time. Symbols: experiment; Lines: calculation.](image-url)
These constants and the coefficients of determination are listed in Table III, and the simulation curves are shown in Figure 2. The maximum rate constant and half-saturation rate coefficient have large ranges (1.394–13797 d\(^{-1}\), and 1.8 \(\times\) 10\(^{-9}\)–99989 g VSS \((g \text{ VS})^{-1}\), respectively). The coefficient of determination for 4\% TS is lower than those of 2 and 8\% TS (Table III).

**Prediction of Kinetic Behaviors of Continuously Mixed Anaerobic Digester with Biological Solid Recycle**

A continuously mixed anaerobic digester with biological solid recycle is a typical operation of wastewater treatment for biogas production. The biological solid recycle enhances the treatment capacity of the completely mixed digester. Lawrence (Lawrence, 1971) derived the relationship between the substrate concentration, as well as the substrate removal efficiency, and the biological solid retention time, \(\theta_c\), for the continuously mixed digester having biological solid recycle at stable state as follows:

\[
\frac{1}{\theta_c} = \frac{Y_{X/S} KSX}{K_s + S} - k_d X
\]  

(12)

Figure 3(a,b) shows plots for 2, 4, 8\% TS using the obtained kinetic parameters in Table III. The substrate masses in effluents decreased and substrate removal

![Fig. 3](image)

**Fig. 3.** Substrate concentrations and substrate removals verse biological solids retention time for completely mixed digester with recycle (Equation (12)). Solid lines: 2\% TS (a); Dot lines: 4\% TS (a); Dash lines: 8\% TS (b).

![Fig. 4](image)

**Fig. 4.** A diagram of completely mixed anaerobic digester with recycle.
efficiencies of digester increased with increasing the biological solid retention time. For the same substrate removal efficiency, a lower TS percentage required a longer biological solid retention time. For example, for 50% substrate removal efficiency at 2 and 4% TS, the biological solid retention times are 41 and 25 days, respectively. However, the HRT changes were not sensitive to changes of substrate concentration for 8% TS. From Figure 3(b), it was found that HRT was almost constant (82 days) when the substrate concentrations in effluents changed to 7.5 mg L$^{-1}$ from 36 mg L$^{-1}$. This is because of the very small value of $K_S$ ($1.6 \times 10^{-8} \ll$substrate concentration) in Table III, which reduces Equation (12) to a linear equation:

$$\frac{1}{Y_c} = (Y_{X/S} K - k_d)X$$ \hspace{1cm} (13)$$

In addition to the stable state operation of digester, unstable state operation also is an interesting topic because the unstable state study involves kinetic behaviors of bioreactors, such as start-up and operating stability of the bioreactor. Figure 4 is a diagram of the continuously mixed digester with biological solid recycle. Two ordinary differential equations could be obtained by calculating the mass balances of VS and VSS in the digester and separator.

$$\frac{dS}{dt} = D(S_0 - S) - \frac{KSX}{K_S + S}$$ \hspace{1cm} (14)$$

$$\frac{dX}{dt} = \frac{F}{V} X_0 - \frac{F_w X_e + F_r X_r}{V} + \frac{Y_{X/S} KSX}{K_S + S} - k_dX$$

$$\hspace{1cm} = DX_0 - DX_e (1 - R_1 + r_1 R_2) + \frac{Y_{X/S} KSX}{K_S + S} - k_dX$$ \hspace{1cm} (15)$$

where $D = V/F$ is the dilution rate (d$^{-1}$), $V$ is the digester volume (L), $F$ is the volumetric flow rate (L d$^{-1}$), $r_1 = X_r/X_e$ is the ratio of VSS concentrations in the recycle flow and the effluent, $R_1 = F_r/F$ is the ratio of volumetric flow rates in the effluent and the influent, and $R_2 = F_r/F_w$ is the ratio of volumetric flow rates in the recycle flow and the waste flow.

If another mass balance of VSS in the separator was examined,

![Graphs](a) Predicted VS (a), VSS (b) in the digester, methane produced (c), and VSS in the exit of separator (d) with respect to time for 2% TS: Solid lines $D = 0$ d$^{-1}$; Dot lines $D = 0.01$ d$^{-1}$; Dash lines $D = 1$ d$^{-1}$. 

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the relationship between VSS at the exit of separator \(X_e\) and VSS \(X\) in the digester could be derived:

\[
X_e = \frac{X(1 + R_1 R_2)}{1 - R_1 + r_1 R_1 + r_1 R_1 R_2} \tag{16}
\]

Substituting Equation (16) into Equation (15) produces a function of VSS in the digester with respect to time:

\[
\frac{dX}{dt} = DX_0 - DX \frac{(1 + R_1 R_2)(1 - R_1 + r_1 R_2)}{1 - R_1 + r_1 R_1 + r_1 R_1 R_2} + \frac{Y_{X/S} K_{S/X}}{K_S + S} \frac{X}{S} \tag{17}
\]

Solving Equations (14 and 17) can produce the trends of VS and VSS with respect to time at unstable state if the parameters, \(r_1\), \(R_1\), and \(R_2\), are selected. Based on the definition of yield, the methane volume \(M_e\) with respect to time also can be predicted by Equation (18):

\[
\Delta M_e = M_e - Y_{P/S} \Delta S = Y_{P/S} (S_0 - S) \tag{18}
\]

where \(Y_{P/S}\) is the yield of product per consumed substrate (mL g\(^{-1}\)) (Table III).

Figures 5–7 are the predicted VS and VSS masses in the digester, methane volumes, and the VSS mass leaving the separator at three dilution rates \(D = 0\) (batch operation), 0.01, and 1 d\(^{-1}\) for 2% TS, \(D = 0\), 0.1, and 10 d\(^{-1}\) for 4% TS, and \(D = 0.1\), 0.2, and 1 d\(^{-1}\) for 8% TS) (assuming \(r_1 = 4\), \(R_1 = 0.1\), and \(R_2 = 1\)) during the startup of the digester without inoculum. It can be found that the effect of dilution rate on these variables is very significant. Increasing the dilution rate, the methane volume and VSS masses in the digester and at the separator exit quickly decreased, while the VS mass in the digester quickly increased. When the dilution rate was equal to zero (batch operation), the

\[\text{Fig. 6. Predicted VS (a), VSS (b) in the digester, methane produced (c), and VSS in the exit of separator (d) with respect to time for 4\% TS: Solid lines } D = 0 \text{ d}^{-1}; \text{ Dot lines } D = 0.1 \text{ d}^{-1}; \text{ Dash lines } D = 10 \text{ d}^{-1}.\]

\[\text{J. Shen and J. Zhu}\]
maximum methane volumes achieved 3071 mL and 4152 mL for 2 and 4\%TS, respectively. When the dilution rates were 1 for 2 and 8\% TS, and 10 for 4\% TS, the VSS values in the effluent were approximately equal to the VSS values in the influent, indicating that a washout situation was approached and the methane production was nearly stopped. Compared to other biochemical reactions, such as the ethanol production from cellulosic materials, the dilution rates to reach washout situations in Figures 5–7 were very low, about one tenth of that in ethanol production at the most (Shen and Agblevor, 2010). This indicated that the anaerobic digester had lower fermentation efficiency than other biochemical reactions, which could be attributed to the low reaction rate of methanogens utilizing the substrate. The time to reach the stable state operation for methane production decreased with increasing the dilution rate, and 2\% TS required a longer time to reach stable state operation than 4 and 8\% TS. There was a drop of VSS at the dilution rate 0 (batch operation) after 90 days of digestion, which might be due to the depletion of substrate as a result of no new substrate added. The simulations also indicated that longer times were needed to reach the maximum methane volumes if without inoculum at the startup of the digester (approximate 140 days for 2\% TS and dilution rate 0, 6 days for 4\% TS and dilution rate 0, and 80 days for 8\% TS and dilution rate 0.1). When the inoculum was added at an amount of 10 times the initial VSS for 2 and 4\% TS, and 1.5 times for 8\% TS, the maximum methane volumes could be achieved in 50, 1 and 40 days at the corresponding dilution rates (Figure 8).

**Fig. 7.** Predicted VS (a), VSS (b) in the digester, methane produced (c), and VSS in the exit of separator (d) with respect to time for 8\% TS: Solid lines $D = 0.1 \text{ d}^{-1}$; Dot lines $D = 0.2 \text{ d}^{-1}$; Dash lines $D = 1 \text{ d}^{-1}$. 

*Modeling Kinetics of Anaerobic Co-digestion of Poultry Litter and Wheat Straw*
Conclusions

Anaerobic co-digestion of poultry litter and wheat straw mixed with municipal wastewater produced the maximum specific methane volume of 209 mL (g initial VS)$^{-1}$ at 50% VSWS in 4% TS. The kinetic parameters of the endogenous decay coefficient, yield coefficient, maximum rate constant, and half-saturation rate coefficient in the Monod model determined by the developed numerical integration using the initial and final VSS data and methane volumes with time in the batch experiments were estimated. The methane volumes, VSS, and VS in the digester and the effluent exiting the separator in the continuously mixed digester system with biological solid recycle could be predicted using the kinetic parameters obtained from the batch experiments. The simulated results showed that the methane volumes produced and the levels of VS and VSS in the digestion operation were greatly affected by the dilution rate. The maximum methane volumes reached were 3071 and 4152 mL for 2 and 4% TS at dilution rate 0, respectively. Using more inoculum at the startup of digester could reduce the digester startup time to reach the maximum methane yield.

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References


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