AN INTRODUCTION TO LAKE AND RESERVOIR WATER QUALITY MODELING

A Short Course Presented at the Arkansas Water Resources Center Annual Spring Conference March 25, 1996

Kenneth H. Reckhow, Ph.D.
Director, Water Resources Research Institute
University of North Carolina
Raleigh, North Carolina

MSC-198
Lake and Reservoir Water Quality Modeling
Kenneth H. Reckhow
March 25, 1997

Topics

Introduction and overview; objectives and outline for lecture (questions and discussion encouraged)

2. What’s the problem? What do we care about? => objectives and attributes

3. Assessment and prediction - scientific analysis in support of decision making => models

4. Model selection criteria
   Mechanistic models (WASP5)

6. Bathtub model

7. Statistical/empirical models (Vollenweider loading criterion; Eutromod)

8. Special topics (as time permits: embayment modeling, uncertainty, trend analysis)
Useful Internet and E-mail Addresses

**US EPA (WASP5)**  
Center for Exposure Assessment Modeling (Athens, GA)  

**US Army Corps of Engineers**  
Waterways Experiment Station (Vicksburg, MS)  
http://www.wes.army.mil/Welcome2.html  
*Bathtub Model:* contact Dr. Robert Kennedy at: kennedr@ex1.wes.army.mil

**Eutromod**  
North American Lake Management Society (NALMS)  
http://www.nalms.org/bkstore/bkstore.htm  
K. Reckhow  
http://www2.ncsu.edu/ncsu/CIL/WRRI/ken's_page.html  
ken_reckhow@ncsu.edu  
reckhow@duke.edu

**General**  
Old Dominion University, Department of Civil and Environmental Engineering  
(Dr. Jaewan Yoon)  
http://www.cee.odu.edu/cee/model/model.html
Model Selection Criteria

- the model is appropriate and comprehensive
- prediction uncertainty is acceptable
- cost and ease of use are reasonable

Model Descriptors/Approaches

- Mechanistic (process oriented)

  Conservation of mass (mass balance):

  \[ \text{Accumulation} = \text{Inputs} - \text{Outputs} \pm \text{Reactions} \]

  \[ V \frac{dP}{dt} = W \ QP \ \sigma PV \]

- Empirical (statistical)

  Statistical (parameter) estimation (e.g., regression)

  \[ Chla = 0.0731P^{1.449} \]
Figure 1.3.2. Model segmentation

Figure 1.4.1  EUTR04 state variable interactions
WASP5 Phytoplankton Growth Equations

\[
Growth \ Rate = G_{\text{max, Temp}} G_{\text{Nutrients}} G_{\text{Light}} = G_{\text{max}}(T) G(I,t) G(N)
\]

\[
G_{\text{max}}(T) = G_{\text{max}}(20 \degree C) \Theta_1(T-20)
\]

where:

\[
\Theta_1 = \text{temperature coefficient, unitless}
\]

\[
G(I,t) = \frac{e}{K_e D} \left[ \exp \left\{ \frac{I_o}{I_s} \exp(-K_e D) \right\} - \exp(-\frac{I_o}{I_s}) \right] \tag{1.4.6}
\]

where

\[
I_s = \frac{G_{\text{max}}(T) \Theta_c e}{\phi_{\text{max}} K_c f_u} \tag{1.4.7}
\]

where

- \( D \) = the average segment depth, m
- \( \phi_{\text{max}} \) = the quantum yield, mg carbon fixed per mole of light quanta absorbed
- \( K_e \) = the total extinction coefficient, computed from the sum of the non-algal light attenuation, \( K_e' \), and the self-shading attenuation due to ambient phytoplankton population, m\(^{-1}\)
- \( K_c \) = the extinction coefficient per unit of chlorophyll, m\(^2\)/mg chlorophyll \( \text{a} \)
- \( f_u \) = units conversion factor (0.083, assuming 43% incident light is visible and 1 mole photons is equivalent to 52,000 cal) mole photons/m\(^2\)-ly
- \( I_o \) = the incident light intensity just below the surface, assumed to average 0.9 I, ly/day
- \( I_s \) = the saturating light intensity of phytoplankton, ly/day
- \( \Theta_c \) = the ratio of carbon to chlorophyll in the phytoplankton, (mg carbon/mg chlorophyll \( \text{a} \))
- \( e \) = the base of natural logarithms (2.71828), unitless

\[
G(N) = \text{Min} \left( \frac{\text{DIN}}{K_{\text{mN}} + \text{DIN}} , \frac{\text{DIP}}{K_{\text{mP}} + \text{DIP}} \right)
\]

DIN, DIP = dissolved nutrient concentrations

\( K_{\text{mN}}, K_{\text{mP}} = \text{Michael-Menten parameters} \)
\( \phi \text{ Nutrients} \)

\( F(l) \)

\( G_m(d^{-1}) \)

Nutrient Concentration (\( \mu g/\lambda \))

Light Intensity (langleys \( d^{-1} \))

Temperature (\( ^\circ C \))
Figure 4.2. Control pathways in empirical eutrophication models developed for CE reservoir applications
SCHEME 1.
SINGLE RESERVOIR, SPATIALLY AVERAGED

SCHEME 2.
SINGLE RESERVOIR, SEGMENTED

SCHEME 3.
PARTIAL RESERVOIR OR EMBAYMENT, SEGMENTED

SCHEME 4.
SINGLE RESERVOIR, SPATIALLY AVERAGED,
MULTIPLE LOADING REGIMES

SCHEME 5.
COLLECTION OF RESERVOIRS, SPATIALLY AVERAGED

SCHEME 6.
NETWORK OF RESERVOIRS, SPATIALLY AVERAGED
a. Morphologic features

b. Segmentation scheme

Figure 4.9. Model segmentation for Lake Keystone, Oklahoma, application
BATHTUB Chlorophyll Equations

Model 1: N, P, Light, Flushing Rate
\[ X_{pn} = \left[ P^{-2} + \frac{(N-150)}{12}\right]^{-0.5} \]
\[ B_x = \frac{X_{pn}^{1.33}}{4.31} \]
\[ G = Z_{mix} \left(0.14 + 0.0039 \cdot F_s\right) \]
\[ B = \frac{C_B \cdot B_x}{(1 + 0.025 \cdot B_x \cdot G) \left(1 + G_a\right)} \]

Model 2: P, Light, Flushing Rate [default]
\[ B_p = \frac{p^{1.37}}{4.88} \]
\[ G = Z_{mix} \left(0.19 + 0.0042 \cdot F_s\right) \]
\[ B = \frac{C_B \cdot B_p}{(1 + 0.025 \cdot B_p \cdot G) \left(1 + G_a\right)} \]

Model 3: P, N, Low-Turbidity
\[ B = C_B \cdot 0.2 \cdot X_{pn}^{1.25} \]

Model 4: P, Linear
\[ B = C_B \cdot 0.28 \cdot P \]

Model 5: Jones and Bachman (1976)
\[ B = C_B \cdot 0.081 \cdot P^{1.46} \]

where:

\( B = \text{chlorophyll } a \text{ concentration (ug/l)} \)
\( P = \text{total phosphorus concentration (ug/l)} \)
\( N = \text{total nitrogen concentration (ug/l)} \)
\( C_B = \text{calibration factor for chlorophyll } a \)
\( Z_{mix} = \text{mean depth of mixed layer (m)} \)
\( F_s = \text{summer flushing rate (yr}^{-1}\text{)} \)
\( G_a = \text{nonalgal turbidity (m}^{-1}\text{)} \)
Figure 4.7. Phosphorus, chlorophyll $a$, and transparency relationships for CE reservoirs
FIG. 8. Chlorophyll and Total Phosphorus Concentrations in the Lake Okeechobee Littoral Zone (May - October Data).
FIG. 3. Next year's mean lake [P] as a function of $(P_i, T_w, z)$ for Okeechobee.

* i.e., predicted from the three-factor multiple regression model
Chl $a = 0.0731P^{1.449}$
Lake Models

Input-Output Models for Nutrients

Mass balance (conservation of mass):

Accumulation = Inputs - outputs ± Reactions

for phosphorus:

\[ \frac{dP}{dt} = W - QP - kPV \]

solution:

\[ P = \frac{W}{Q+kV} \left[ 1 - e^{-\frac{1}{k+1}t} \right] + P_i e^{-\frac{1}{k+1}t} \]

steady-state solution:

\[ P = \frac{W}{Q+kV} = \frac{P_{in}}{1+k\tau} \]

other similar expressions:

\[ P = \frac{L}{v_s+q_s} = P_{in}(1 - R_p) \]

- \( V \) = lake volume
- \( P \) = phosphorus concentration
- \( W \) = phosphorus mass loading
- \( Q \) = volumetric water load
- \( k \) = first order reaction rate
- \( \tau \) = water residence time
- \( P_{in} \) = influent
- \( p \) = concentration
- \( v_s \) = apparent settling
Eutromod Equations for Arkansas Lakes

**Total Phosphorus (mg/l)**

\[
\log_{10}(P) = \log_{10}\left( \frac{P_{in}}{1 + k\tau} \right)
\]

where: \( k = 10.77\tau^{-0.61}z^{0.01}P_{in}^{0.82} \)

**Total Nitrogen (mg/l)**

\[
\log_{10}(N) = \log_{10}\left( \frac{N_{in}}{1 + k\tau} \right)
\]

where: \( k = 0.46\tau^{-0.75}z^{0.22}N_{in}^{0.95} \)

**Chlorophyll a (ug/l)**

\[
\log_{10}(chla) = 1.99 + 0.5\log_{10}\left( \hat{P} \right) + 0.23\log_{10}(\tau) - 0.351\log_{10}(z)
\]

**Secchi Disk Depth (m)**

\[
\log_{10}(SD) = -1.32 - 0.66\log_{10}\left( \hat{P} \right) + 0.47\log_{10}(z)
\]

where:
- \( \tau \) = water residence time
- \( z \) = mean depth
- \( \hat{} \) ~ “predicted”
- \( in \) ~ influent
EUTROMOD
Version 2.50
by
Kenneth H. Reckhow
Duke University
Durham, NC 27706
1991

A watershed/lake modeling procedure for eutrophication management, with region-specific models and with emphasis on uncertainty analysis.

Enter two letter (all caps) state postal code (e.g., NY) in box, identifying the location of the lake.

Hit ALT-X to continue; this selects the region-specific lake models.
## Eutromod:
### Map of Screen Worksheets

<table>
<thead>
<tr>
<th>20</th>
<th>Surface water runoff</th>
<th>USLE</th>
<th>Land areas</th>
<th>Lake depth</th>
<th>Detention time, etc.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Precipitation</td>
<td></td>
<td>Septic tanks</td>
<td>Treatment plants</td>
<td>Water runoff</td>
</tr>
<tr>
<td></td>
<td>USLE</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>Phosphorus concentrations in inputs</td>
<td>Nitrogen concentrations in inputs</td>
<td>Attenuation zones for nutrient trapping</td>
<td>Calculation tables</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>Phosphorus-total loading by land use category</td>
<td>Nitrogen-total loading by land use category</td>
<td>Attenuation zones for nutrient trapping</td>
<td>Calculation tables</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>Lake response predictions (uncertainty due to hydrologic variability)</td>
<td>Lake response predictions (uncertainty due to model error)</td>
<td>Attenuation zones for nutrient trapping</td>
<td>Calculation tables</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>Allowable nutrient loading (chlor a goal)</td>
<td>Allowable nutrient loading (chlor a, P goals)</td>
<td>Calculations-dissolved nutrients</td>
<td></td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>Allowable nutrient loading (TSI goal)</td>
<td></td>
<td>Calculations-sediment-attached nutrients</td>
<td></td>
<td></td>
</tr>
<tr>
<td>140</td>
<td></td>
<td></td>
<td>Calculations-total nutrients</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 Letters across the top of the table and numbers along the left side identify cells in the spreadsheet (e.g., the intersection of "T" and "60" is cell T60).
Figure A.2

Surface Water Runoff & Soil Loss

Figure A.3

Universal Soil Loss Equation

<table>
<thead>
<tr>
<th>Land Use Category</th>
<th>LS factor</th>
<th>C factor</th>
<th>P factor</th>
<th>Xi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture1</td>
<td>0.3</td>
<td>0.2</td>
<td>0.5</td>
<td>5.263</td>
</tr>
<tr>
<td>Agriculture2</td>
<td>0.4</td>
<td>0.03</td>
<td>0.3</td>
<td>0.631</td>
</tr>
<tr>
<td>Agriculture3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Agriculture4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Agriculture5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Forest</td>
<td>0.9</td>
<td>0.001</td>
<td>0.2</td>
<td>0.031</td>
</tr>
<tr>
<td>Urban1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Urban2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Feedlots</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Other1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Other2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Other3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

X (total soil loss) = 2.12098961 t/ha (area-weighted average)
### Figure A.6

**Phosphorus Concentration Estimates**

<table>
<thead>
<tr>
<th>Land Use Category</th>
<th>Dissolved</th>
<th>Sed-Attach</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture1</td>
<td>0.07</td>
<td>220</td>
<td>*****</td>
</tr>
<tr>
<td>Agriculture2</td>
<td>0.07</td>
<td>220</td>
<td>*****</td>
</tr>
<tr>
<td>Agriculture3</td>
<td>0</td>
<td>0</td>
<td>*****</td>
</tr>
<tr>
<td>Agriculture4</td>
<td>0</td>
<td>0</td>
<td>*****</td>
</tr>
<tr>
<td>Agriculture5</td>
<td>0</td>
<td>0</td>
<td>*****</td>
</tr>
<tr>
<td>Forest</td>
<td>0.008</td>
<td>220</td>
<td>*****</td>
</tr>
<tr>
<td>Urban1</td>
<td>*****</td>
<td>*****</td>
<td>0.2</td>
</tr>
<tr>
<td>Urban2</td>
<td>*****</td>
<td>*****</td>
<td>0.1</td>
</tr>
<tr>
<td>Feedlots</td>
<td>*****</td>
<td>*****</td>
<td>0</td>
</tr>
<tr>
<td>Other1</td>
<td>0</td>
<td>0</td>
<td>*****</td>
</tr>
<tr>
<td>Other2</td>
<td>0</td>
<td>0</td>
<td>*****</td>
</tr>
<tr>
<td>Other3</td>
<td>0</td>
<td>0</td>
<td>*****</td>
</tr>
<tr>
<td>Precipitation</td>
<td>*****</td>
<td>*****</td>
<td>0.05</td>
</tr>
</tbody>
</table>

\[P\text{-enrichment ratio} = 2\]

### Figure A.7

**Nitrogen Concentration Estimates**

<table>
<thead>
<tr>
<th>Land Use Category</th>
<th>Dissolved</th>
<th>Sed-Attach</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture1</td>
<td>2.8</td>
<td>500</td>
<td>*****</td>
</tr>
<tr>
<td>Agriculture2</td>
<td>1.8</td>
<td>500</td>
<td>*****</td>
</tr>
<tr>
<td>Agriculture3</td>
<td>0</td>
<td>0</td>
<td>*****</td>
</tr>
<tr>
<td>Agriculture4</td>
<td>0</td>
<td>0</td>
<td>*****</td>
</tr>
<tr>
<td>Agriculture5</td>
<td>0</td>
<td>0</td>
<td>*****</td>
</tr>
<tr>
<td>Forest</td>
<td>0.19</td>
<td>500</td>
<td>*****</td>
</tr>
<tr>
<td>Urban1</td>
<td>*****</td>
<td>*****</td>
<td>1.5</td>
</tr>
<tr>
<td>Urban2</td>
<td>*****</td>
<td>*****</td>
<td>1.75</td>
</tr>
<tr>
<td>Feedlots</td>
<td>*****</td>
<td>*****</td>
<td>0</td>
</tr>
<tr>
<td>Other1</td>
<td>0</td>
<td>0</td>
<td>*****</td>
</tr>
<tr>
<td>Other2</td>
<td>0</td>
<td>0</td>
<td>*****</td>
</tr>
<tr>
<td>Other3</td>
<td>0</td>
<td>0</td>
<td>*****</td>
</tr>
<tr>
<td>Precipitation</td>
<td>*****</td>
<td>*****</td>
<td>0.1</td>
</tr>
</tbody>
</table>

\[N\text{-enrichment ratio} = 2\]
### Phosphorus Loading Estimates - By Category

<table>
<thead>
<tr>
<th>Category</th>
<th>Expected Loading (kg/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>546.93439</td>
</tr>
<tr>
<td>Forest</td>
<td>7.2419592</td>
</tr>
<tr>
<td>Urban</td>
<td>19.635</td>
</tr>
<tr>
<td>Feedlots</td>
<td>0</td>
</tr>
<tr>
<td>Precipitation</td>
<td>37.6635</td>
</tr>
<tr>
<td>Septic Tanks</td>
<td>24.375</td>
</tr>
<tr>
<td>Point Sources</td>
<td>0</td>
</tr>
<tr>
<td>Other</td>
<td>0</td>
</tr>
</tbody>
</table>

Estimated Total = 635.849855 (kg/yr)

### Nitrogen Loading Estimates - By Category

<table>
<thead>
<tr>
<th>Category</th>
<th>Expected Loading (kg/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>9847.0767</td>
</tr>
<tr>
<td>Forest</td>
<td>145.88448</td>
</tr>
<tr>
<td>Urban</td>
<td>158.6865</td>
</tr>
<tr>
<td>Feedlots</td>
<td>0</td>
</tr>
<tr>
<td>Precipitation</td>
<td>75.327</td>
</tr>
<tr>
<td>Septic Tanks</td>
<td>216.125</td>
</tr>
<tr>
<td>Point Sources</td>
<td>0</td>
</tr>
<tr>
<td>Other</td>
<td>0</td>
</tr>
</tbody>
</table>

Estimated Total = 10443.0997 (kg/yr)

### Predicted Lake Trophic State Variables - Based on Model Uncertainties (Median Values)

<table>
<thead>
<tr>
<th>Variable (units)</th>
<th>-1 Std Err</th>
<th>Predicted</th>
<th>+1 Std Err</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total P-in (mg/l)</td>
<td>1.3226</td>
<td>0.8085</td>
<td>0.0577</td>
</tr>
<tr>
<td>Total N-in (mg/l)</td>
<td>0.0259</td>
<td>0.0387</td>
<td>1.1544</td>
</tr>
<tr>
<td>Total P (mg/l)</td>
<td>0.6860</td>
<td>0.8899</td>
<td>0.8282</td>
</tr>
<tr>
<td>Total N (mg/l)</td>
<td>0.3964</td>
<td>0.5187</td>
<td></td>
</tr>
<tr>
<td>Chlor a (ug/l)</td>
<td>12.7905</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Secchi Depth (m)</td>
<td>1.4660</td>
<td>1.0826</td>
<td></td>
</tr>
<tr>
<td>Prob Hypo Anoxia</td>
<td></td>
<td>0.3964</td>
<td></td>
</tr>
<tr>
<td>Prob BG Dominant</td>
<td></td>
<td>0.5187</td>
<td></td>
</tr>
<tr>
<td>THMs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TSI</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>